Do Vortices Entangle?

Cynthia J. Olson Reichhardt, T-12, and Matthew B. Hastings, T-13

The high superconducting transition temperatures of compounds such as $YBa_2Cu_3O_{7-\delta}$ (YBCO) lead to a very rich set of behaviors of the magnetic vortex lines that form inside the superconducting material in the presence of an applied magnetic field. In particular, thermal fluctuations of the vortex lines can produce a vortex melting transition. The nature of the molten vortex state has been a subject of intense debate for nearly two decades. Since there can be significant thermally induced wiggling along the length of the vortex in the liquid state, D. Nelson proposed that neighboring vortex lines may become entangled with each other, much like polymers in a melt. Similarly to the entangled polymer state, the effective viscosity of the entangled vortex state is expected to increase significantly. This could explain the striking experimentally observed increase in critical current upon increasing temperature.

In order for the vortices to entangle, it is crucial that neighboring flux lines cannot cut through each other easily and reconnect into a disentangled state. Theoretical estimates of the cutting barrier range over two orders of magnitude in energy, and span both the regime where cutting is impossible as well as the regime where cutting occurs easily. Numerical simulations have proven similarly ambiguous, with some simulations interpreted as providing evidence for entanglement and others interpreted as showing that the lines cut and do not entangle. It is therefore natural to turn to experiments to resolve the issue. Unfortunately, experimental evidence for or against entanglement based on bulk measurements has also proven ambiguous. Thus, despite more than a decade of theoretical, numerical, and experimental studies, the question of whether vortices can form an entangled state has not yet been convincingly answered.

We propose a direct experimental test of vortex entanglement by means of a local magnetic force microscope (MFM) probe, which can unambiguously determine whether it is possible for two vortices to wind around each other without cutting. We consider a high-temperature superconductor containing two closely spaced magnetic dots at the bottom of sample, shown in Fig. 1(a). A mobile magnetic dot is introduced to the top of the sample in the form of a magnetic MFM tip (open circle). The magnetic dots attract the vortices, so that the top and bottom positions of the vortices are fixed at the dot locations. As the MFM is moved along the surface of the sample, it drags the top of one vortex with a directly measurable force. When the MFM tip moves in a circular path, the two vortices wind together, producing the entangled state illustrated in Fig. 1(b). As the winding angle increases, the force required to drag the vortex further around increases according to a form that we derive theoretically. If the vortices cut, this force will abruptly drop. Thus, using such an experiment, it is possible to directly probe whether vortex entanglement can occur.

The magnitude of the angular force is large enough to be detected experimentally. For two vortices in a YBCO sample, we estimate that the maximum force should be approximately 28 pN; forces as small as 0.4 pN have been measured previously with MFM techniques. In more highly anisotropic materials such as $BiSr_2Ca_2CuO_8$, the elastic line model for vortices assumed in D. Nelson's work is expected to break down and vortex cutting should occur immediately. In this case, the force required to move the top portion of one vortex around the second vortex is two orders of magnitude smaller than the entanglement force in YBCO, placing it below the threshold of detection by MFM.

The local experimental probe that we propose can also be used to explore numerous other properties of the vortex system besides vortex entanglement. For example, in a geometry containing only one magnetic pin and one vortex line, the MFM tip can be used to measure the vortex line tension directly. If the line tension is known, the tip could be used to tear a vortex away from an individual pinning site, such as a grain boundary, and the pinning force could be measured. Local rheology measurements are also possible in the vortex lattice state; for example, the local elastic constants can be probed by moving a single vortex back and forth around its lattice equilibrium position. The temperature dependence of both the elastic constants and the pinning energy could also be probed.

We have proposed an experimental setup for constructing and probing entangled states of superconducting vortices, and shown that the forces associated with vortex entanglement are experimentally measurable. This kind of experimental setup can be generalized to other types of vortices, as in fluid turbulence. Within the elastic string model, we find that the entangled state is stable only up to a maximum pitch or minimum vortex spacing. The instabilities we have found raise the question of whether the entangled state can exist with a high density of vortices [1]. To answer this question and to compare pancake and elastic string models, an experimental test of our proposal is desirable.

For more information contact Cynthia J. Olson Reichhardt at cjrx@lanl.gov.

[1] C.J. Olson Reichhardt and M.B. Hastings, *Phys. Rev. Lett.* **92**, 157002 (2004).

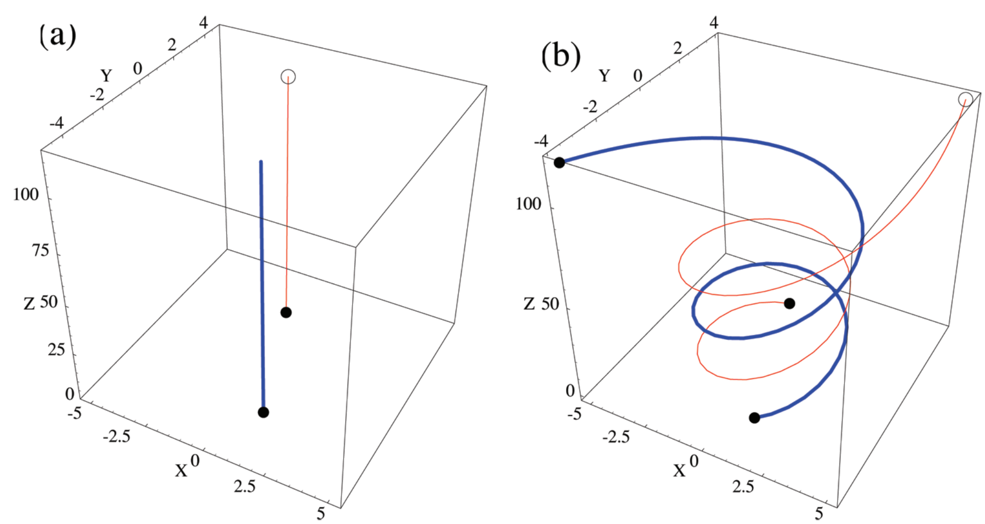

Fig. 1. *Artificial creation of an entangled vortex state. (a) Schematic of the starting state showing fixed pins (black dots) and the magnetic force microscope tip (open circle). (b) The entangled configuration produced by winding the top of one vortex around the other.*